# Evaluation of Distributed Detection Methods for of Agile Targets

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# Likelihood Ratio Test

The Sequential Likelihood Ratio (LR) test is a statistically optimal algorithm to decide between two hypotheses:

- $h_1$: there is a target.
- $h_0$: there is no target.

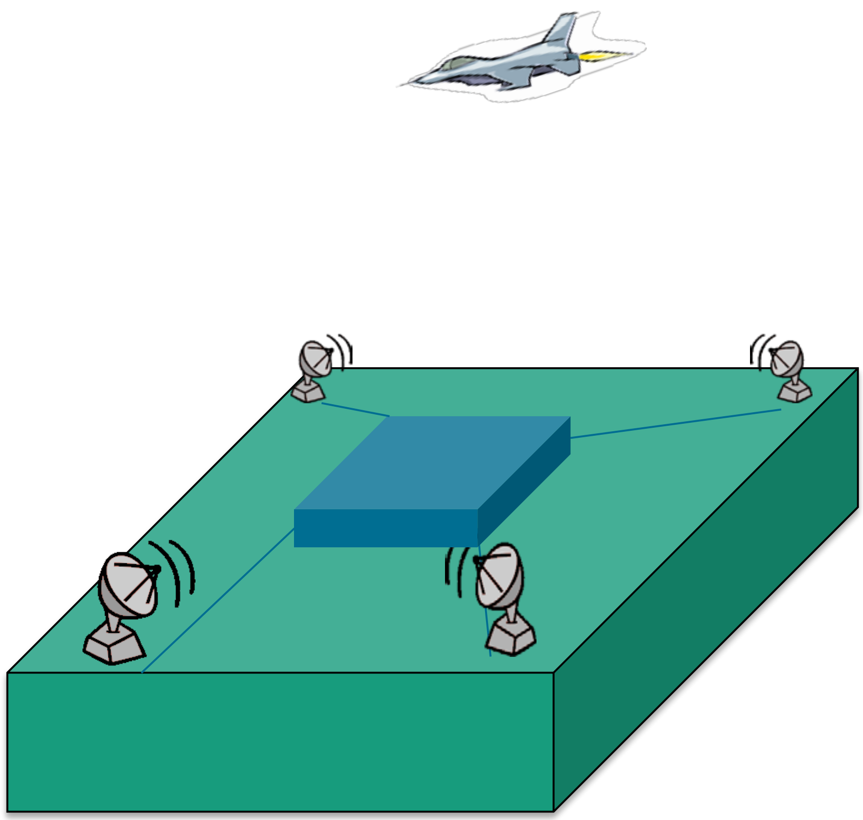
The decision is inferred from measurements $\mathcal{Z}^k = \{\mathcal{Z}_1^k, \ldots, \mathcal{Z}_S^k\}$

A decision is based on the iterative rule for $k$=1,2,3,…

$$\mathrm{LR}(k) = \frac{p(h_1|\mathcal{Z}^k)}{p(h_0|\mathcal{Z}^k)}$$

- $\mathrm{LR}(k) < A$: accept $h_0$, i.e. delete track
- $\mathrm{LR}(k) > B$: accept $h_1$, i.e. confirm track
- $A < \mathrm{LR}(k) < B$: continue processing.

# Track Confirmation
## with overlapping Field of View



- **Naïve:**
  - Local tracks are accepted as confirmed
- **Local Decision Fusion:**
  - Weighted Mean of local existence probability
- **Centralized:**
  - All sensor data is transmitted to a Fusion Center (FC)
- **Distributed**
  - All sensors send sufficient statistics ($\bar{p}^s$) to the FC

# Recursive computation of the LR

According to Bayes, one has

$$\mathrm{LR}(k) = \frac{p(\mathcal{Z}^k|h_1)}{p(\mathcal{Z}^k|h_0)}$$

where

$$p(\mathcal{Z}^k|h_i) = p(Z_k|\mathcal{Z}^{k-1}, h_i)\, p(\mathcal{Z}^{k-1}|h_i)$$

$$\overset{i=1}{=} \int \mathrm{d}\mathbf{x}_k\, p(Z_k|\mathbf{x}_k, h_1)\;\; p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)\, p(\mathcal{Z}^{k-1}|h_1)$$

$$\mathrm{LR}(k) = \mathrm{LR}(k-1) \cdot \Lambda(k)$$

$$\Lambda(k) = \frac{\int \mathrm{d}\mathbf{x}_k\, p(Z_k|\mathbf{x}_k, h_1)\, p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)}{p(Z_k|h_0)}$$

# Distributed Kalman Filter (DKF)

The Distributed Kalman Filter (DKF) is based on the *Product Representation:*

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \propto \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right)$$

For *non-linear scenarios*, the **Federated Kalman Filter (FKF)** needs to be used, since the measurement model is data dependent.

One obtains an approximation:

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) \approx \frac{1}{c_{k|k-1}} \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}^s_{k|k-1}, \mathbf{P}^s_{k|k-1}\right)$$

# Distributed Sequential Likelihood Ratio Test

The LR score at time *k* is given by

$$\mathrm{LR}(k) = \mathrm{LR}(k-1) \cdot \Lambda(k)$$

$$\Lambda(k) = \frac{\int \mathrm{d}\mathbf{x}_k \, p(Z_k|\mathbf{x}_k, h_1) \, p(\mathbf{x}_k|\mathcal{Z}^{k-1}, h_1)}{p(Z_k|h_0)}$$

where

$$p(Z_k|h_0) = |\mathrm{FoV}|^{-m} p_F(m)$$

$$p(Z_k|\mathbf{x}_k, h_1) = (|\mathrm{FoV}|^{-m} p_F(m))((1-p_D) + \frac{p_D}{\rho_F} \sum_{j=1}^{m} \mathcal{N}(\mathbf{z}_j;\, \mathbf{H}_k\mathbf{x}_k,\, \mathbf{R}_k))$$

$$p(\mathbf{x}_k|\mathcal{Z}^{k-1}) = \frac{1}{c_{k|k-1}} \prod_{s=1}^{S} \mathcal{N}(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1})$$

Now, the normalization constant is important!

Stochastic sensor model



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# DKF Normalization Constant

The normalization constant is given by

$$c_{k|k-1} = \int \mathrm{d}\mathbf{x}_k \prod_{s=1}^{S} \mathcal{N}\left(\mathbf{x}_k;\, \mathbf{x}^s_{k|k-1},\, \mathbf{P}^s_{k|k-1}\right).$$

Algebraic manipulations yield

Closed form solution for the integral!

$$c_{k|k-1} = \prod_{s=1}^{S-1} \mathcal{N}\left(\mathbf{x}^{s+1}_{k|k-1};\, \mathbf{x}^{(1:s)}_{k|k-1},\, \mathbf{P}^{(1:s)}_{k|k-1} + \mathbf{P}^s_{k|k-1}\right)$$

$$\mathbf{x}^{(1:s)}_{k|k-1} = \mathbf{P}^{(1:s)}_{k|k-1} \sum_{i=1}^{s} (\mathbf{P}^i_{k|k-1})^{-1} \mathbf{x}^i_{k|k-1},$$

$$\mathbf{P}^{(1:s)}_{k|k-1} = \left(\sum_{i=1}^{s} (\mathbf{P}^i_{k|k-1})^{-1}\right)^{-1}.$$

# Sequential LR Update for DKF

The updating factor of the LR is given by

$$\Lambda(k) = \frac{1}{c_{k|k-1}} \int \mathrm{d}\mathbf{x}_k \prod_{s=1}^{S} \Bigg\{ \Big( (1 - p_D) + \frac{p_D}{\rho_F} \sum_{j=1}^{m_s} \mathcal{N}\big(\mathbf{z}_k^{j,s};\, \mathbf{H}_k^s \mathbf{x}_k,\, \mathbf{R}_k^s\big) \Big) \cdot \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k-1}^s,\, \mathbf{P}_{k|k-1}^s\big) \Bigg\}$$

Standard KF/EKF for each sensor node yields

unnormalized hypotheses weights

$$\Lambda(k) = \frac{1}{c_{k|k-1}} \int \mathrm{d}\mathbf{x}_k \left\{ \prod_{s=1}^{S} \sum_{j=0}^{m_s} p^{\star\, j,s}\, \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k}^{j,s},\, \mathbf{P}_{k|k}^{j,s}\big) \right\}$$

# Normalization and Moment Matching

The normalized weights are given by

$$p^{j,s} = \frac{p^{\star\, j,s}}{\bar{p}^s}$$

$$\bar{p}^s = \sum_{j=0}^{m_s} p^{\star\, j,s}.$$

Therefore:

$$\sum_{j=0}^{m_s} p^{\star\, j,s}\, \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{j,s}, \mathbf{P}_{k|k}^{j,s}\right) = \bar{p}^s \underbrace{\sum_{j=0}^{m_s} p^{j,s}\, \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{j,s}, \mathbf{P}_{k|k}^{j,s}\right)}$$

$$\overset{\text{MM}}{\approx} \mathcal{N}\left(\mathbf{x}_k; \mathbf{x}_{k|k}^{s}, \mathbf{P}_{k|k}^{s}\right)$$



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# Computation of Lambda

As a result one obtains

$$\Lambda(k) = \frac{c_{k|k}}{c_{k|k-1}} \prod_{s=1}^{S} \bar{p}^s \int \mathrm{d}\mathbf{x}_k \; \mathcal{N}\big(\mathbf{x}_k;\, \mathbf{x}_{k|k},\, \mathbf{P}_{k|k}\big)$$

$$= \frac{c_{k|k}}{c_{k|k-1}} \prod_{s=1}^{S} \bar{p}^s$$

where the posterior normalization constant is given by

$$c_{k|k} = \prod_{s=1}^{S-1} \mathcal{N}\big(\mathbf{x}_{k|k}^{s+1};\, \mathbf{x}_{k|k}^{(1:s)},\, \mathbf{P}_{k|k}^{(1:s)} + \mathbf{P}_{k|k}^{s}\big)$$

# Conclusion: Distributed Track Existence Decision

## Local Sensor Nodes

Prediction: Relaxed Evolution Model

$$\mathbf{x}^s_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{x}^s_{k|k-1},$$
$$\mathbf{P}^s_{k|k-1} = \mathbf{F}_{k|k-1}\mathbf{P}^s_{k|k-1}\mathbf{F}^\top_{k|k-1} + S\,\mathbf{Q}_{k|k-1}$$

Filtering:

- Update state parameters with EKF / MHT / PDAF / …
- calculate decision contribution

$$p^{\star\, j,s} = \begin{cases} (1-p_D) \\ \frac{p_D}{\rho_F}\mathcal{N}\left(\mathbf{z}^{j,s}_k; \mathbf{H}^s_k\mathbf{x}^s_{k|k-1}, \mathbf{S}^s_k\right) \end{cases}$$

$$\bar{p}^s = \sum_{j=0}^{m_s} p^{\star\, j,s}.$$

Tx:

$$\mathbf{x}^s_{k|k}$$
$$\mathbf{P}^s_{k|k}$$
$$\bar{p}^s$$

## Fusion Center

Prediction: calculate prior constants using the Relaxed Evolution Model and the previous transmission:

$$c_{k|k-1} = \prod_{s=1}^{S-1}\mathcal{N}\left(\mathbf{x}^{s+1}_{k|k-1}; \mathbf{x}^{(1:s)}_{k|k-1}, \mathbf{P}^{(1:s)}_{k|k-1} + \mathbf{P}^s_{k|k-1}\right),$$

Filtering: calculate posterior constants using the new transmissions. Update LR score:

$$\mathrm{LR}(k) = \Lambda(k)\cdot\mathrm{LR}(k-1)$$

$$\Lambda(k) = \frac{c_{k|k}}{c_{k|k-1}}\prod_{s=1}^{S}\bar{p}^s$$

# NUMERICAL EVALUATION

# Simulation Setup

For the evaluation, a realistic multi-radar scenario has been chosen:

- 4 radars arranged along a circle of about 13 km radius
- Poisson distributed FA with mean 5 per sensor per scan
- The target, if present, has a process noise of psd = 10
- Probability of detection is $p_D = 0.2, 0.5$ and $p_D = 0.9$.
- A no target scenario is also considered
- We compare against:
  - Centralized processing (CKF) for LR calculation (optimal)
  - Decentralized mean of all local LR scores (LKF)



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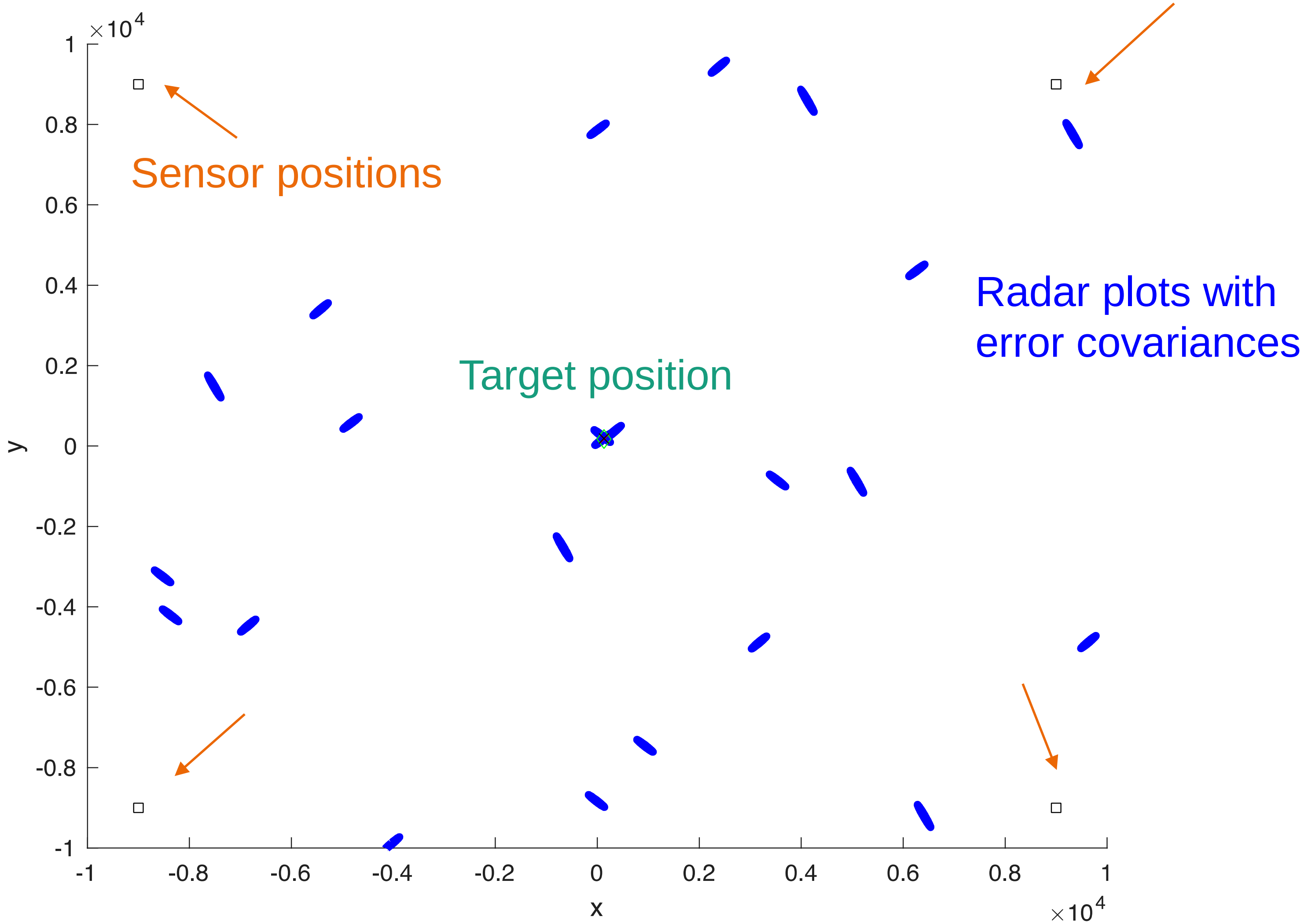
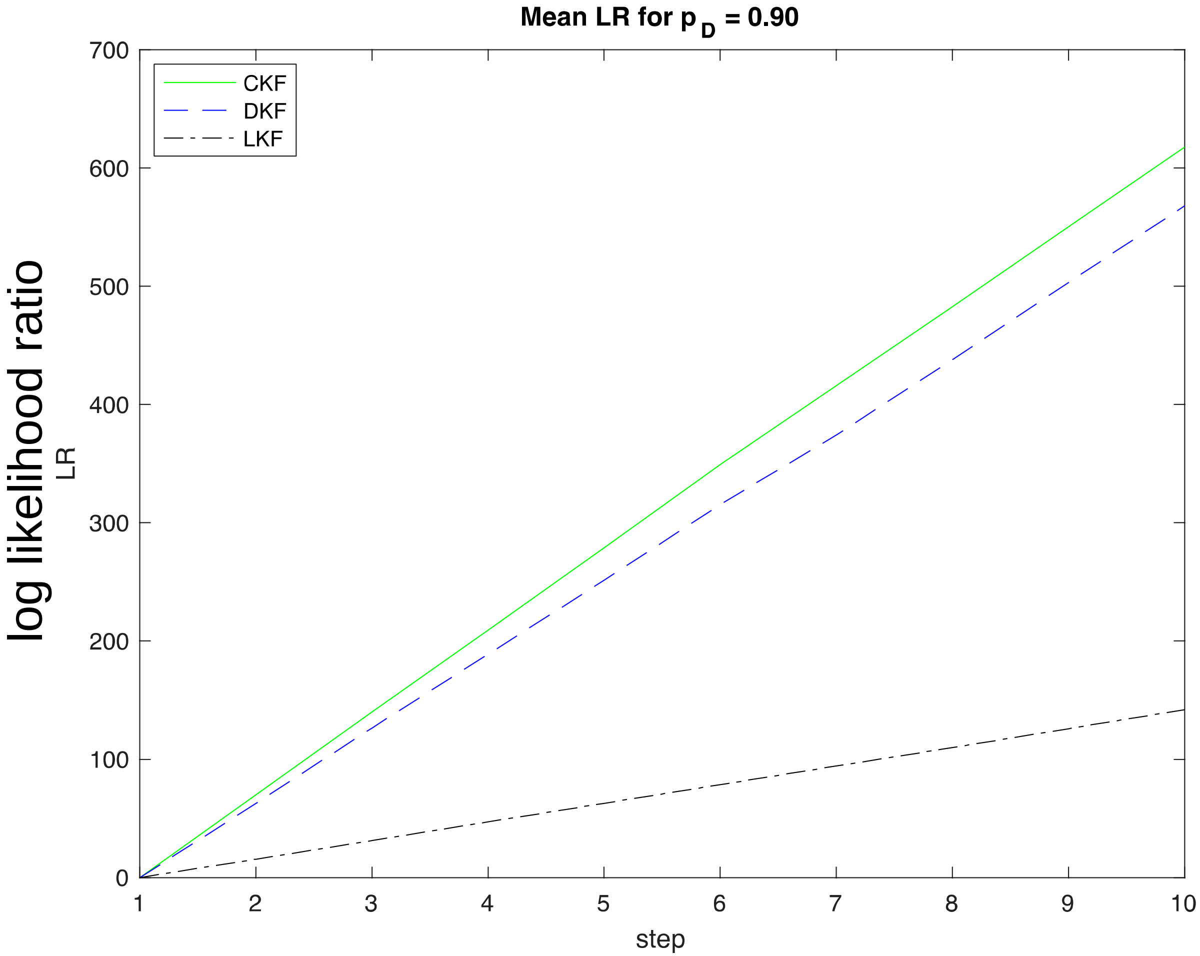
# Plot of a single scan



# Numerical Results of the LR Scores

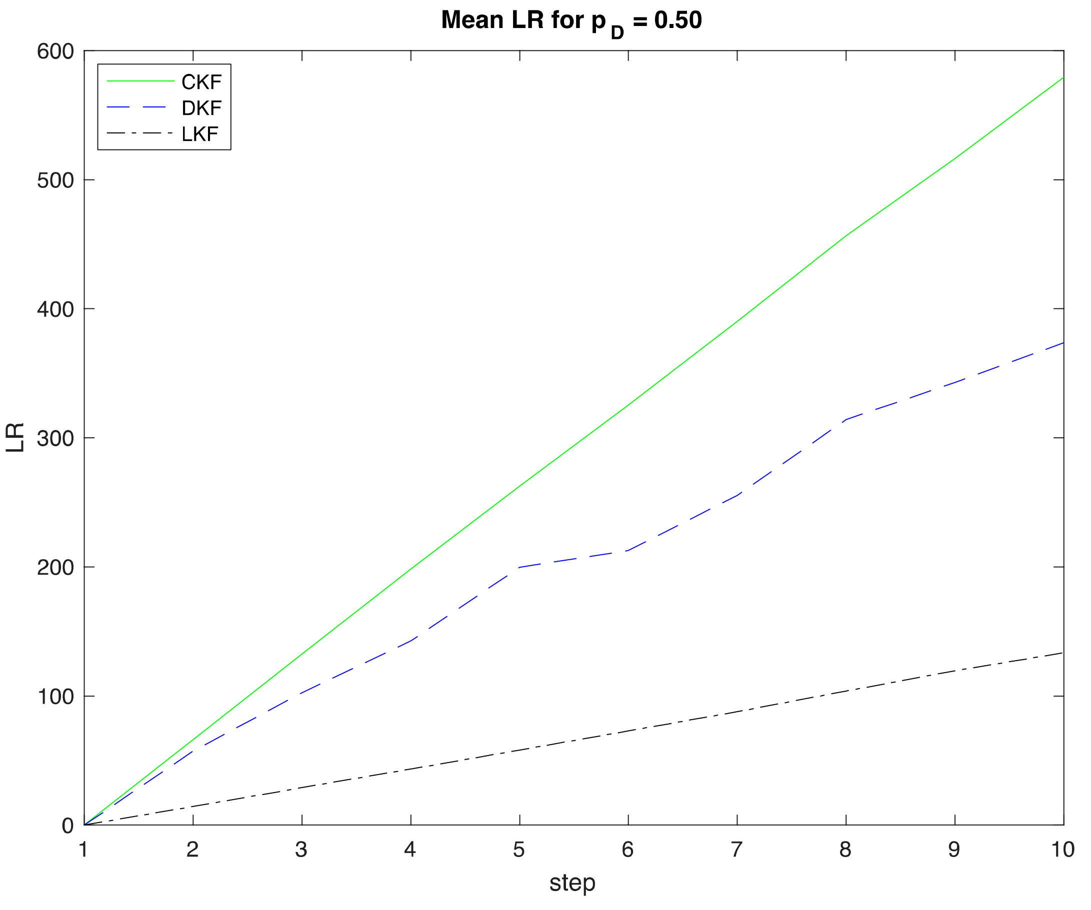
$p_D = 0.9$



# Numerical Results of the LR Scores

$$p_D = 0.5$$

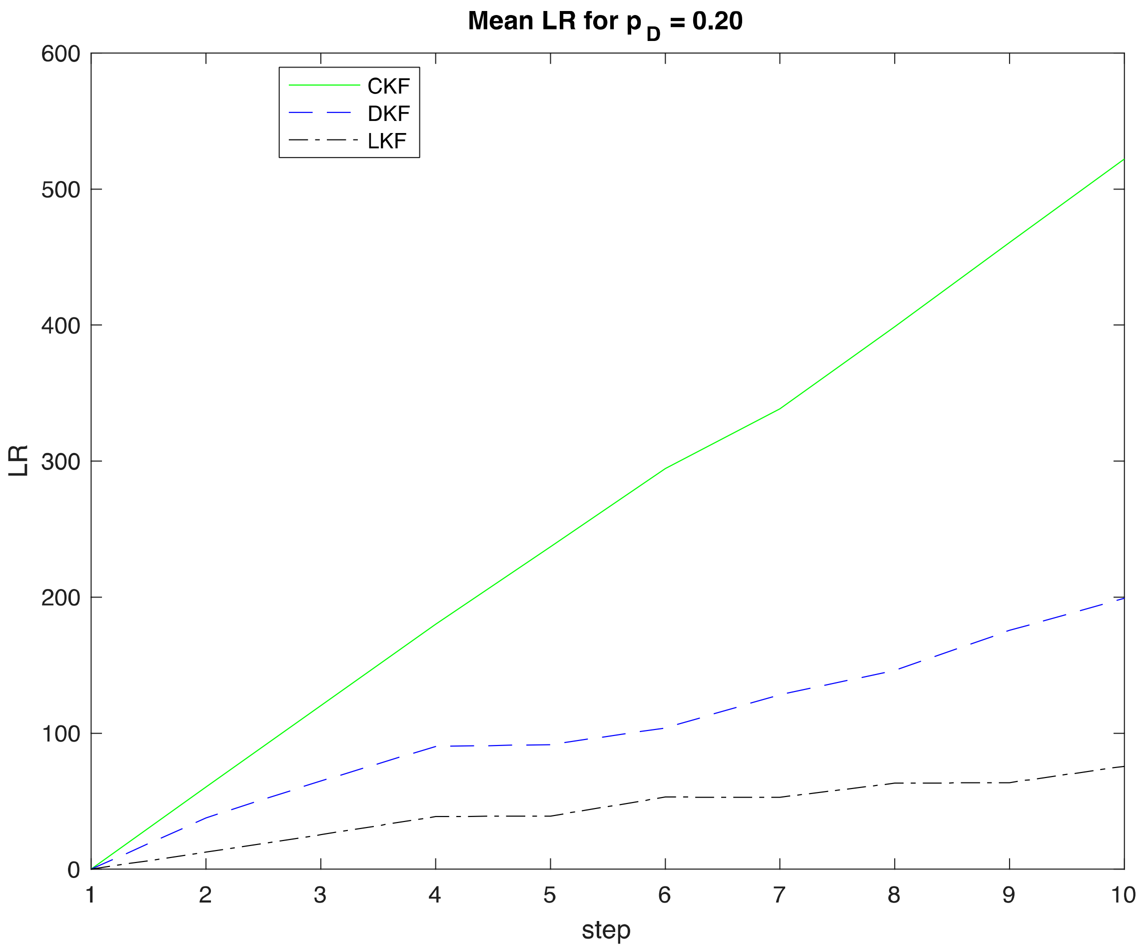
log likelihood ratio



# Numerical Results of the LR Scores

$$p_D = 0.2$$

log likelihood ratio



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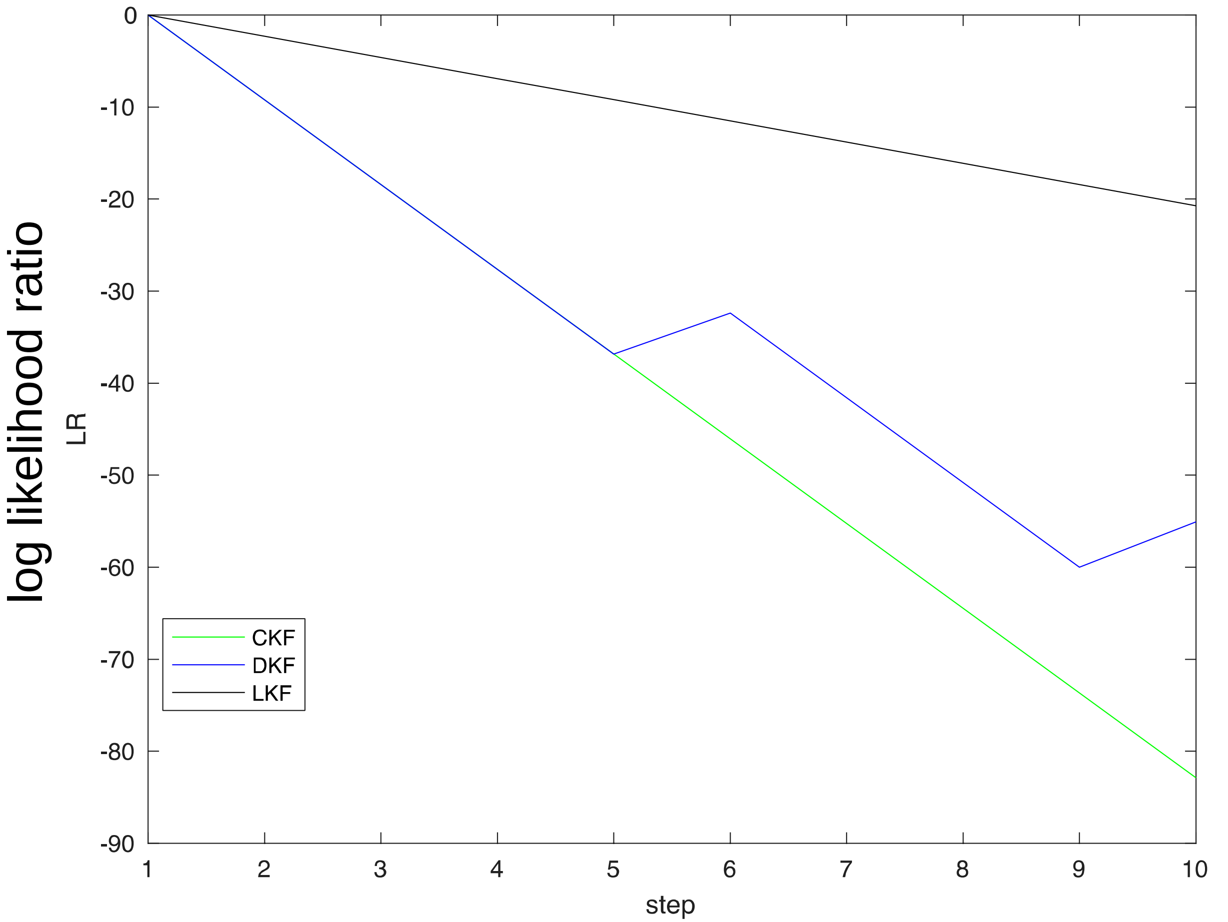
# Numerical Results of the LR Scores

No target



# Conclusions

- Sequential Likelihood Ratio test for Federated Kalman filter in non-linear applications was derived.
- Fusion center computes LR score based on single reel valued parameter from each sensor.
- The distributed calculation clearly performs better than averaging the local LR scores even with identical sensors parameters.

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